Permafrost melt and its effects on planetary energy balance

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Mathematics and Climate Seminar

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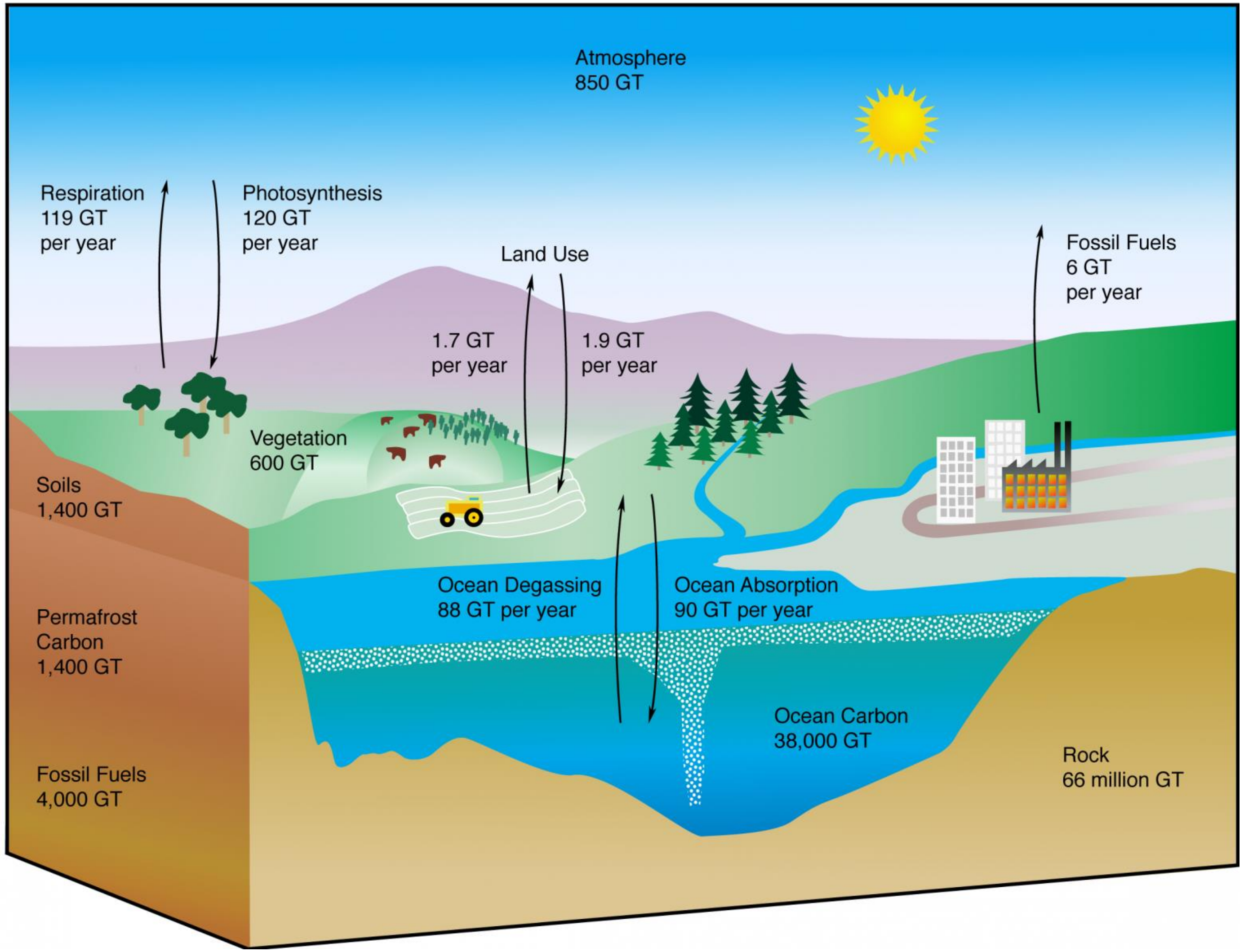
Motivation: the role of permafrost in global energy balance



(NSIDC)

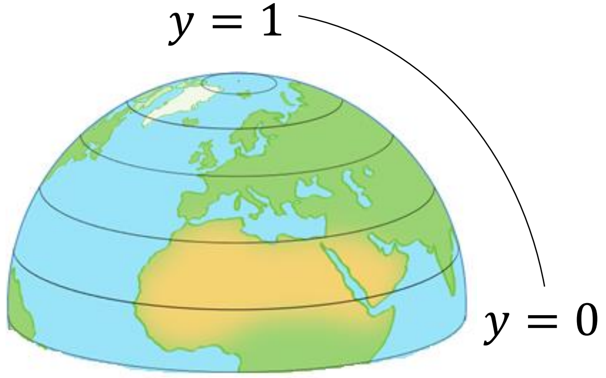
Surface Temperature: Budyko's energy balance model

Model surface energy balance using temperature:

$$R\,\frac{\partial T}{\partial t} = \big(1 - \underbrace{\alpha(y,\eta)}_{\text{albedo}}\big)\underbrace{Qs(y)}_{\substack{\text{incoming}\\\text{solar}\\\text{radiation}}} \;-\; \underbrace{(A + BT)}_{\substack{\text{outgoing}\\\text{longwave}\\\text{radiation}}} \;+\; \underbrace{C(\bar{T} - T)}_{\substack{\text{heat}\\\text{transport}}}$$

$y = \sin(\text{latitude})$

$\eta = \text{ice line}$



$$\alpha(y,\eta) = \begin{cases} \alpha_1\ , & y > \eta \quad [\text{ice}] \\ \alpha_2\ , & y < \eta \quad [\text{not ice}] \end{cases}$$

$$s(y) \approx 1 + s_2\,(3y^2 + 1)$$

(Budyko 69, North 75)

Soil Temperature: Heat conduction

At each latitude, we assume temperature varies by depth via conduction:

$$\frac{\partial T_y}{\partial t} = k \frac{\partial^2 T_y}{\partial z^2}, \qquad t \geq 0, \qquad 0 \leq z \leq L$$

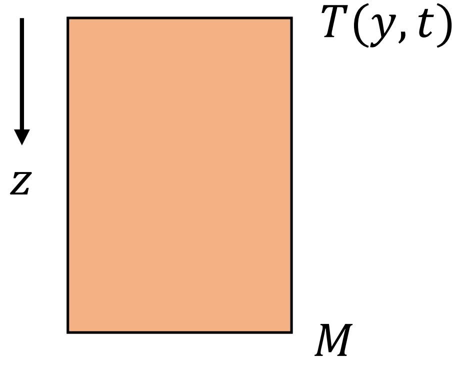
$y = \sin(\text{latitude})$

$z =$ soil depth

At surface boundary: $T_y(t, 0) = T(t)$

At the lower boundary: $T_y(t, L) = M$

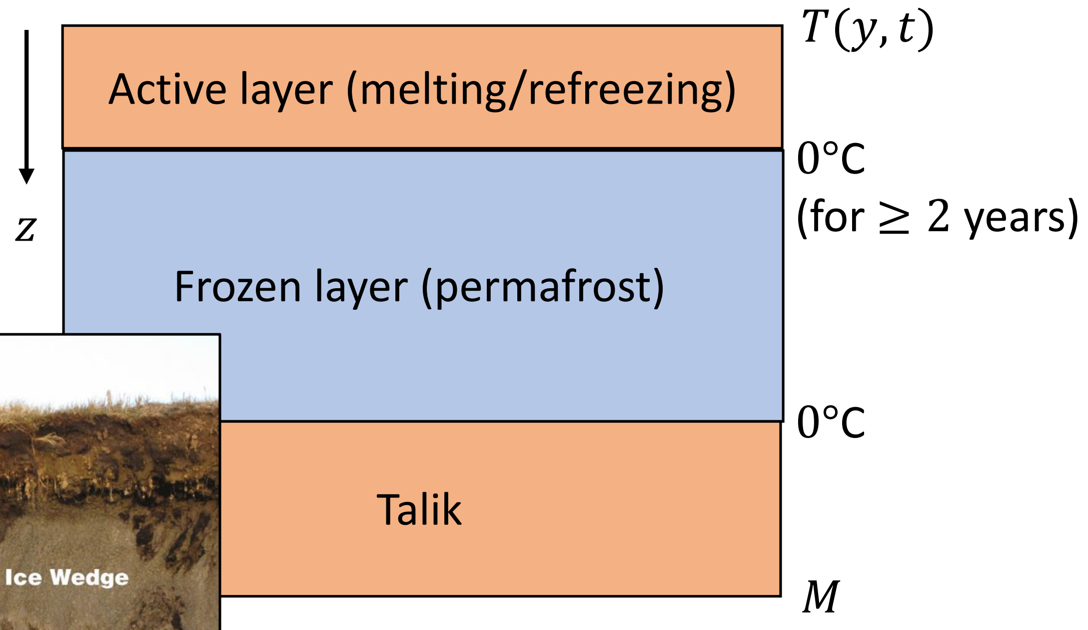
Initial condition: $T_y(0, z) = \frac{M - T(0)}{L} z + T(0)$

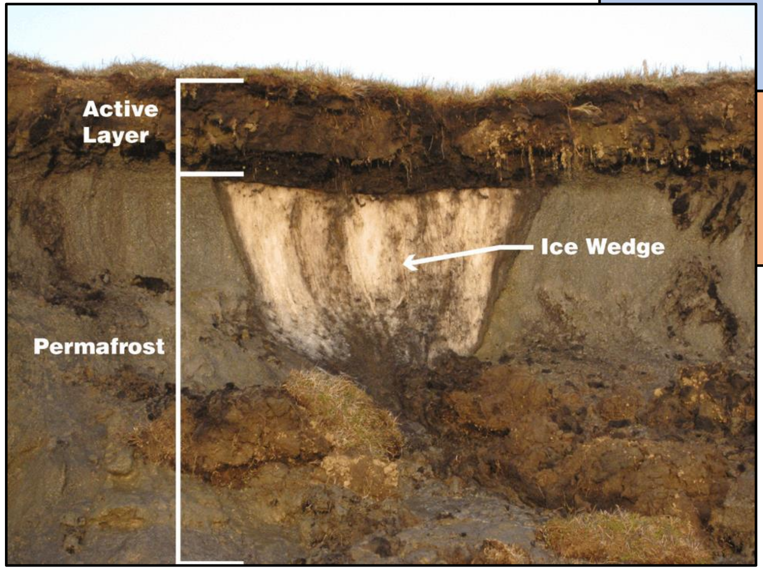


Heat conduction as a model for permafrost

Permafrost is soil that has been frozen for at least two years:

- Frozen soil/ice composite
- $\leq 0^\circ$C for at least two years
- Active layer:
  - top portion melting/refreezing
- Maximum depth:
  - 500 m (modern) – 1,000 m (paleo)





Heat conduction as a model for permafrost

Thermal diffusivity:

$$k = \frac{\alpha}{\rho c}$$

Thermal conductivity (α)

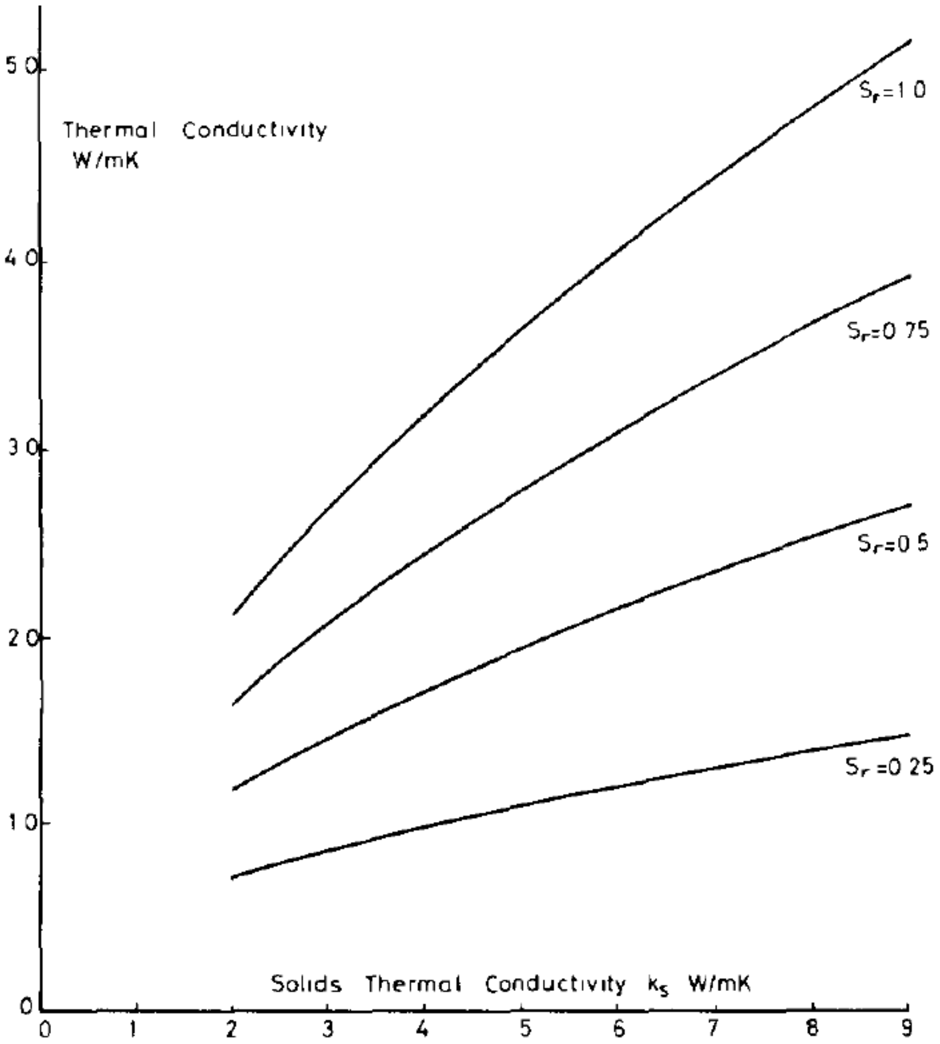
Volumetric heat capacity (ρc)

$k \in (75, 828)$ m²/yr

Crust heat source:

$M \approx 30$°C (at L $\approx 1{,}000$ m)

(Farouki 81, Lachenbruch 86)



S_r : saturation (%)

Heat conduction as a model for permafrost

Is it reasonable to model permafrost as the heat equation?

On a decadal timescale, yearly variations may be important:

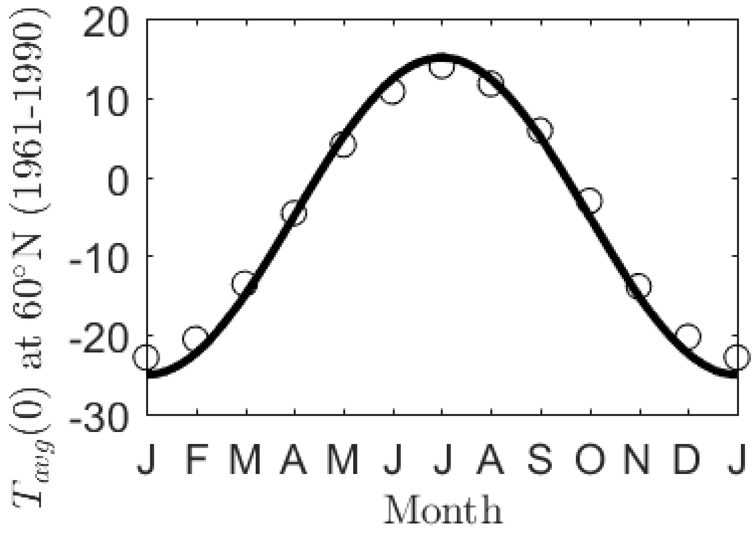
$$\frac{\partial T_y}{\partial t} = k\frac{\partial^2 T_y}{\partial z^2}, \qquad t \geq 0, \qquad 0 \leq z \leq L$$

$$T_y(0,t) = T(y,t) \approx (-5 - 20\cos(2\pi t))$$

At 61°N

$$T_y(l,t) = M$$

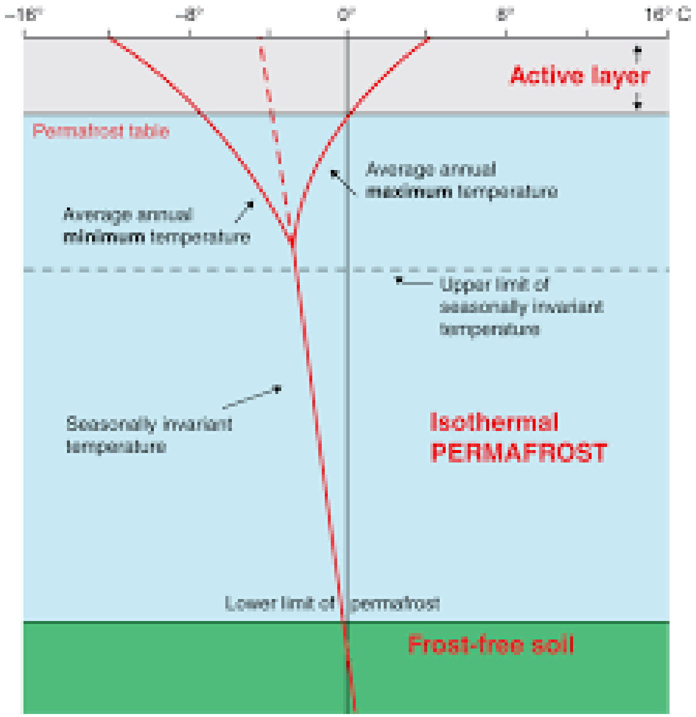
$$T_y(z,0) = \frac{M - T(y,0)}{l} z + T(y,0)$$

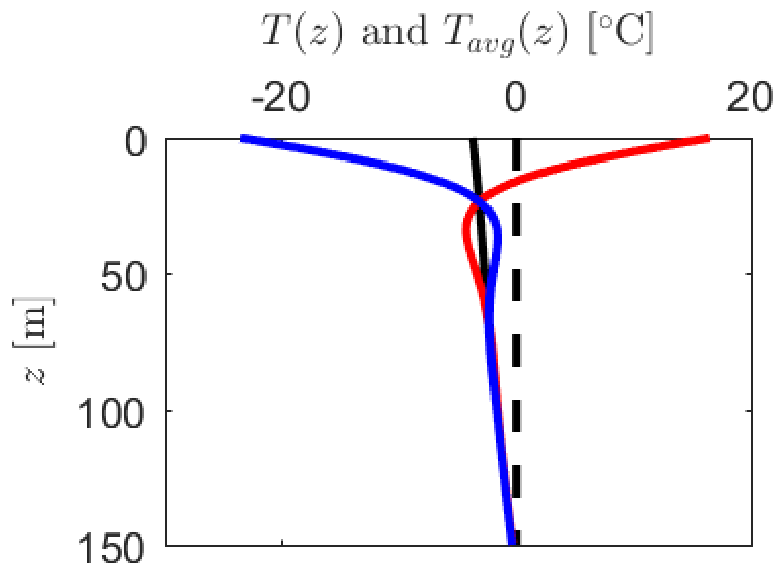


(data: CRU CL v2.0)

Heat conduction as a model for permafrost

The temperature profile has similar characteristics to permafrost:





(Sandells and Flocco)

Heat conduction as a model for permafrost

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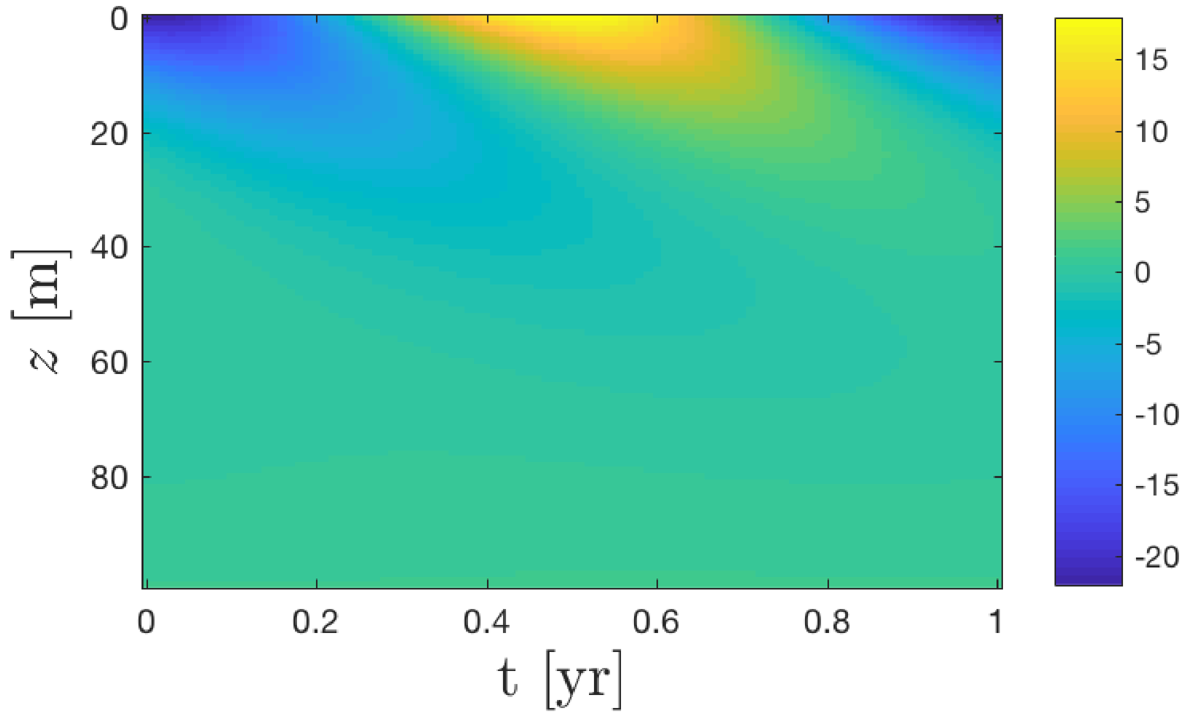
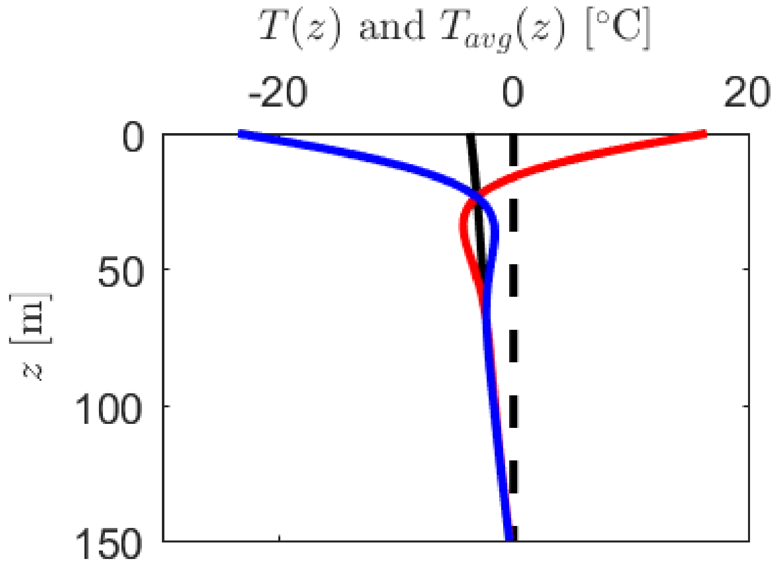




Heat conduction as a model for permafrost

With added forcing, we can simulate the permafrost melting:

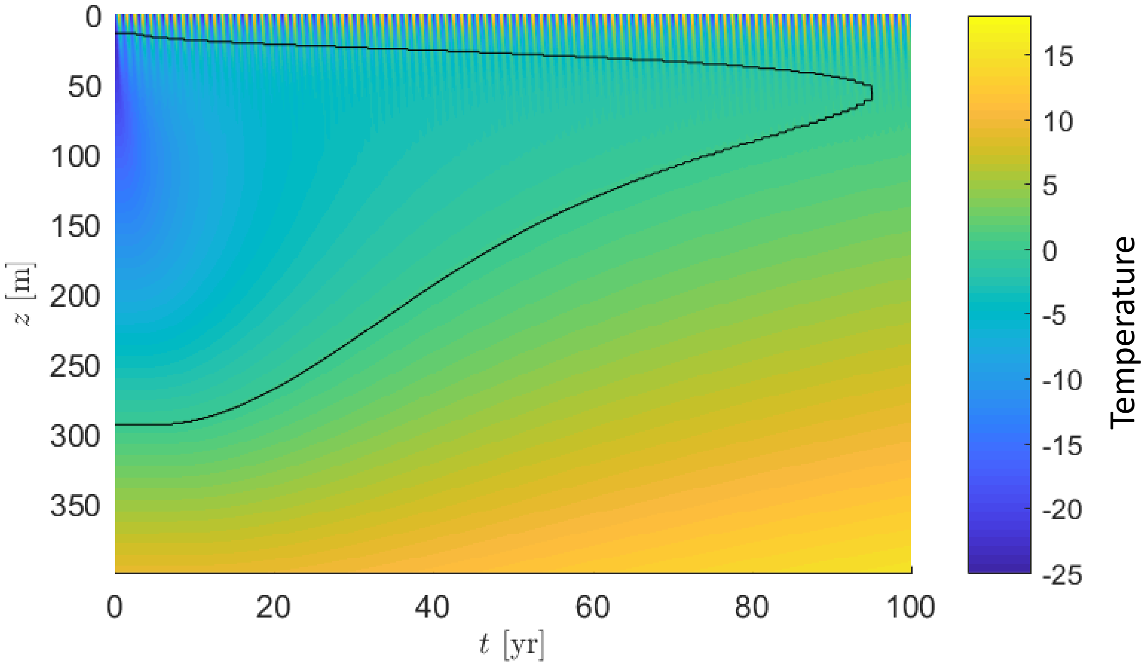
$$\frac{\partial T_y}{\partial t} = k \frac{\partial^2 T_y}{\partial z^2}, \qquad t \geq 0, \qquad 0 \leq z \leq L$$

$$T_y(0,t) = T(y,t) + F(t) \approx (-5 - 20\cos(2\pi t)) + \frac{F_{max} t}{t_{max}}$$

$$T_y(l,t) = M$$

$$T_y(z,0) = \frac{M - T(y,0)}{l} z + T(y,0)$$

$F_{max} = 3°\text{C}$



$(k = 700,\ M = 60,\ L = 1{,}000)$

Heat conduction as a model for permafrost

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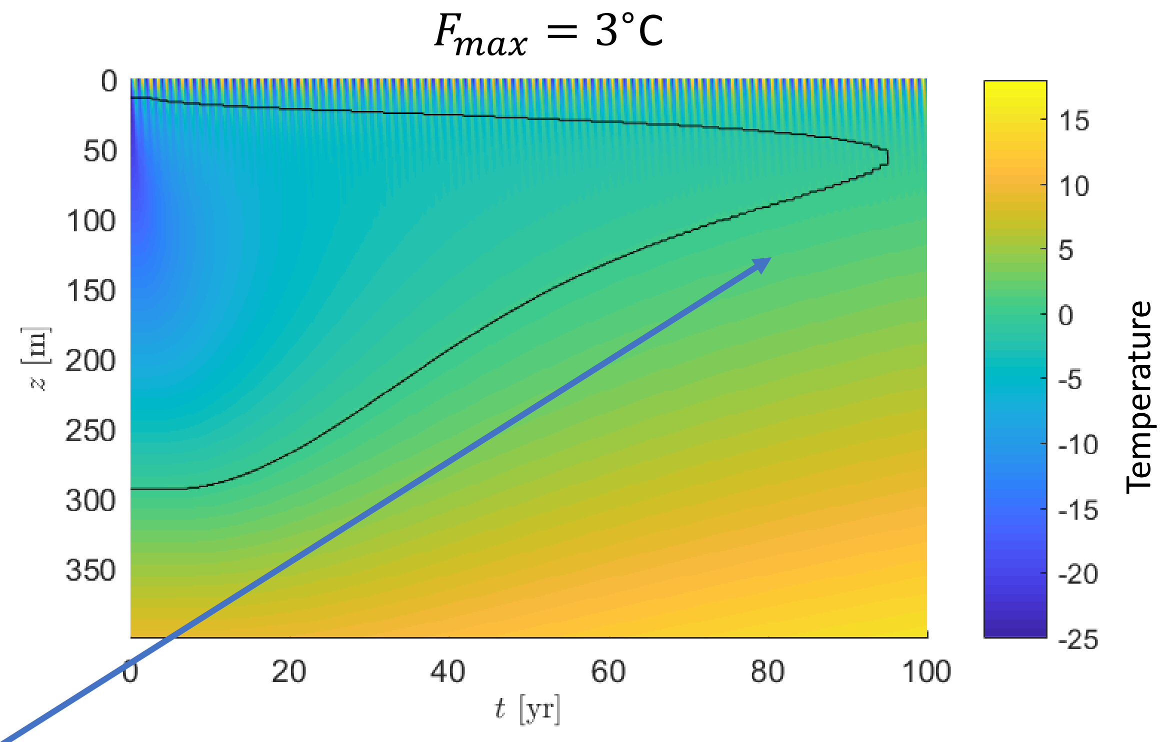


Signal for permafrost craters?

$(k = 700,\ M = 60,\ L = 1{,}000)$

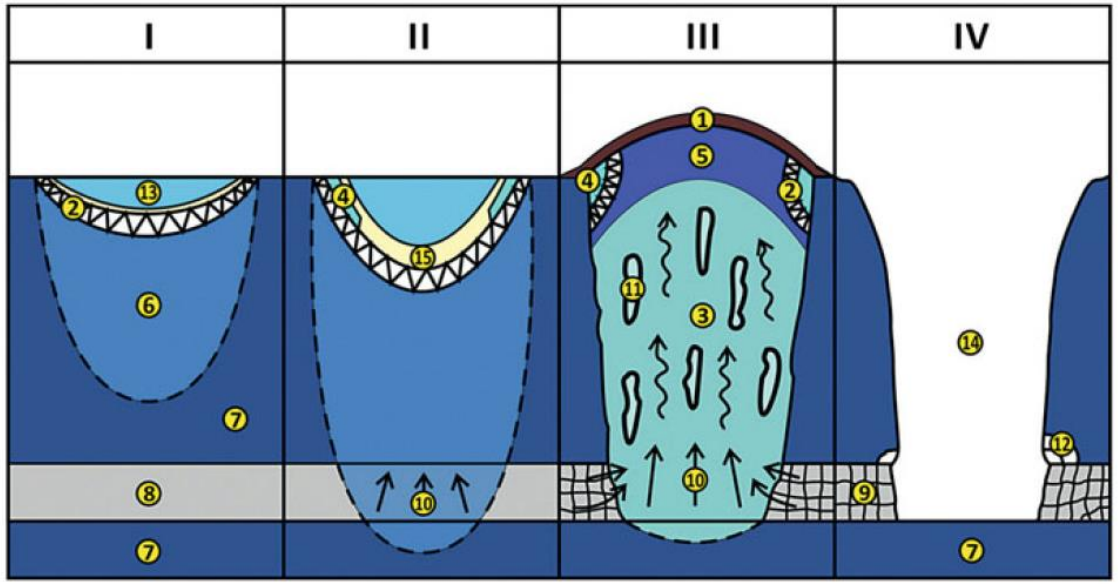
Heat conduction as a model for permafrost

Since 2014, multiple observations of permafrost craters in Siberia (Yamal Peninsula +)

(Leibman et al 14)

Heat conduction as a model for permafrost

Several studies have followed:



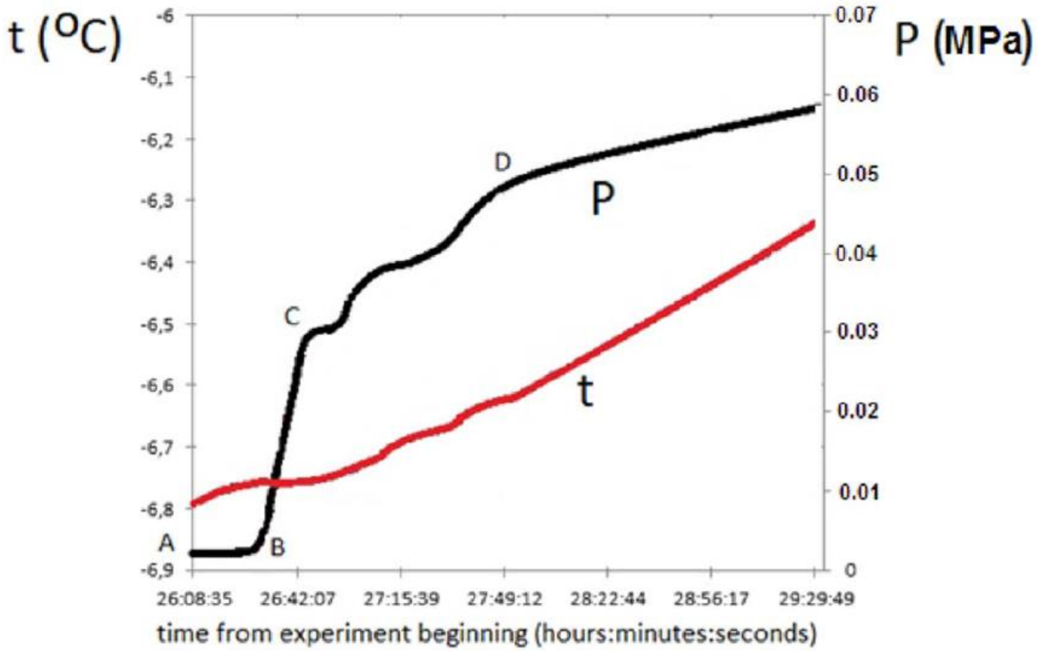


Fig. 4. Experimental curves of pressure (P) and temperature (t) changes inside the cell when slow heating of hydrate-containing sample.

(Khimenkov 19)

(Yakushev 18)

Coupling Budyko's model to the heat equation

Together, the system is given by:

$$R\,\frac{\partial T}{\partial t} = \big(1 - \alpha(y,\eta)\big)Qs(y) \; - \; (A + BT) \; + \; C(\bar{T} - T)$$

$$\frac{\partial T_y}{\partial t} = k\,\frac{\partial^2 T_y}{\partial z^2}$$

Let

$A = A_1 + A_2 z_{\text{melt}},$ $\qquad$ A_2 - average rate of increase of surface temperature in regions with permafrost

$\approx$ 1.4 to 2.0, per 3 meters melted (from above)

(Schneider von Deimling 12)

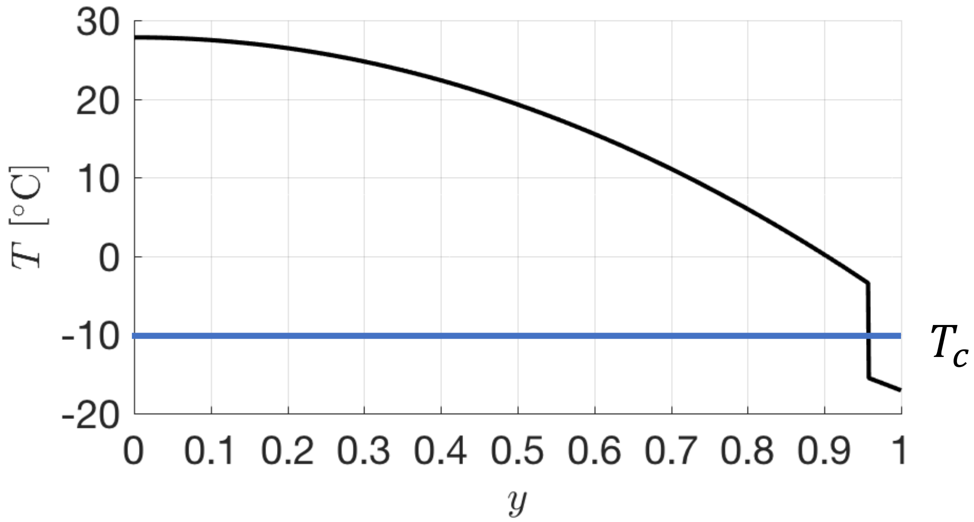
Without permafrost amplification

In steady state, equilibria of the system are given by

$$T(y) = \frac{1}{B+C}\Big(Qs(y)\big(1-\alpha(y)\big) - A_1 + C\bar{T}\Big)$$

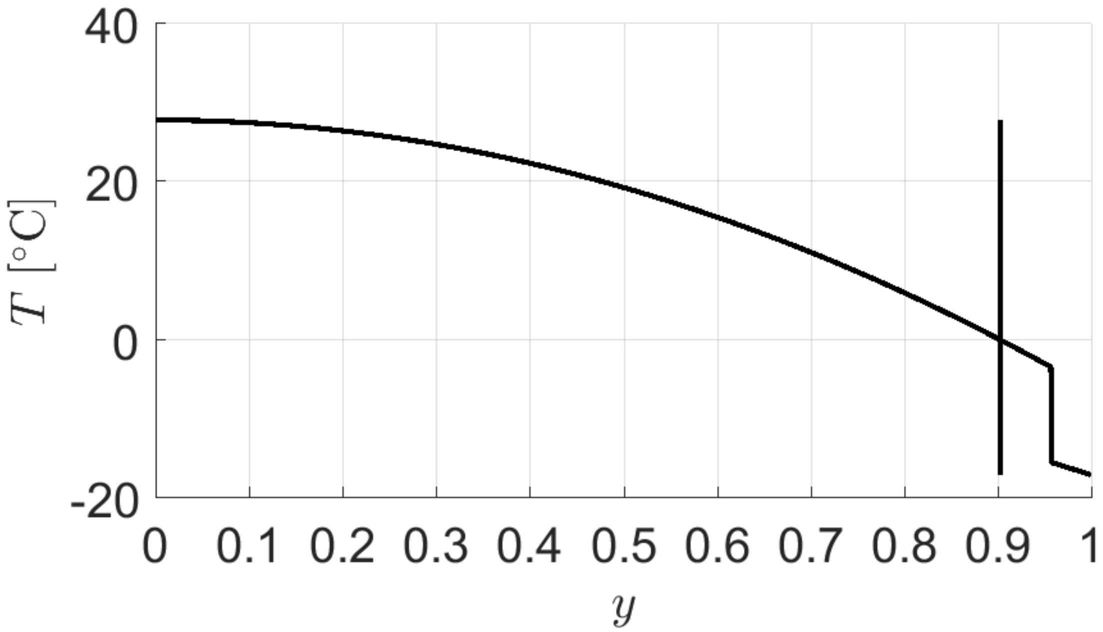
$$= \begin{cases} \frac{1}{B+C}(Qs(y)(1-\alpha_1) - A_1 + C\bar{T}), & T > T_c \\ \frac{1}{B+C}(Qs(y)(1-\alpha_2) - A_1 + C\bar{T}), & T < T_c \end{cases}$$

$$T_y(z) = \frac{M - T(y)}{l} z + T(y)$$



Without permafrost amplification

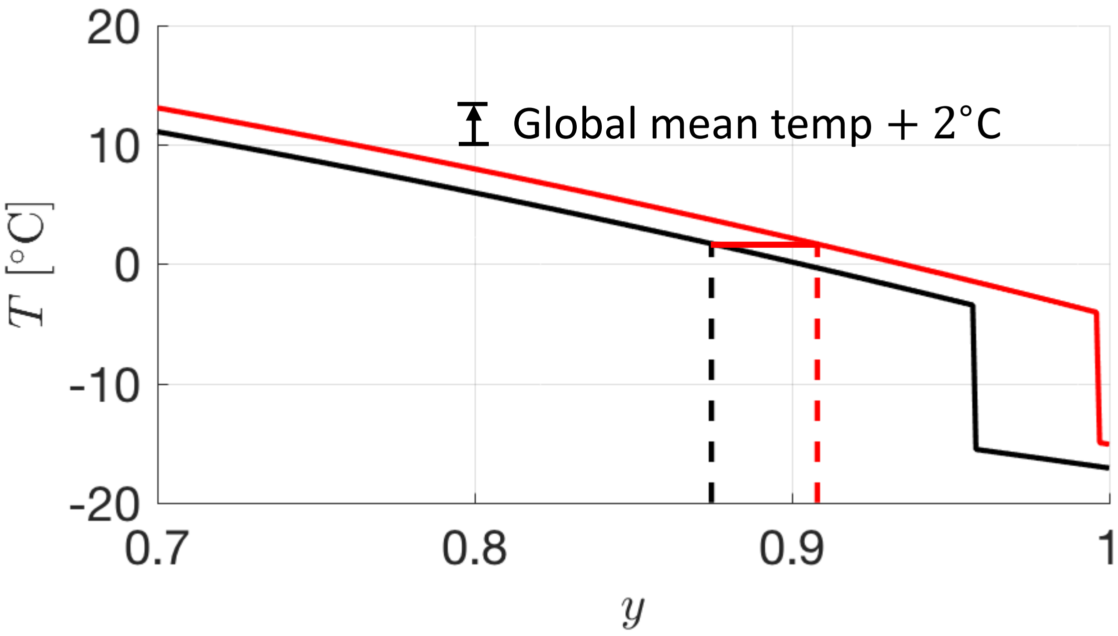
If we include the lowest equilibrium latitude where $T_y(0) = 0$:



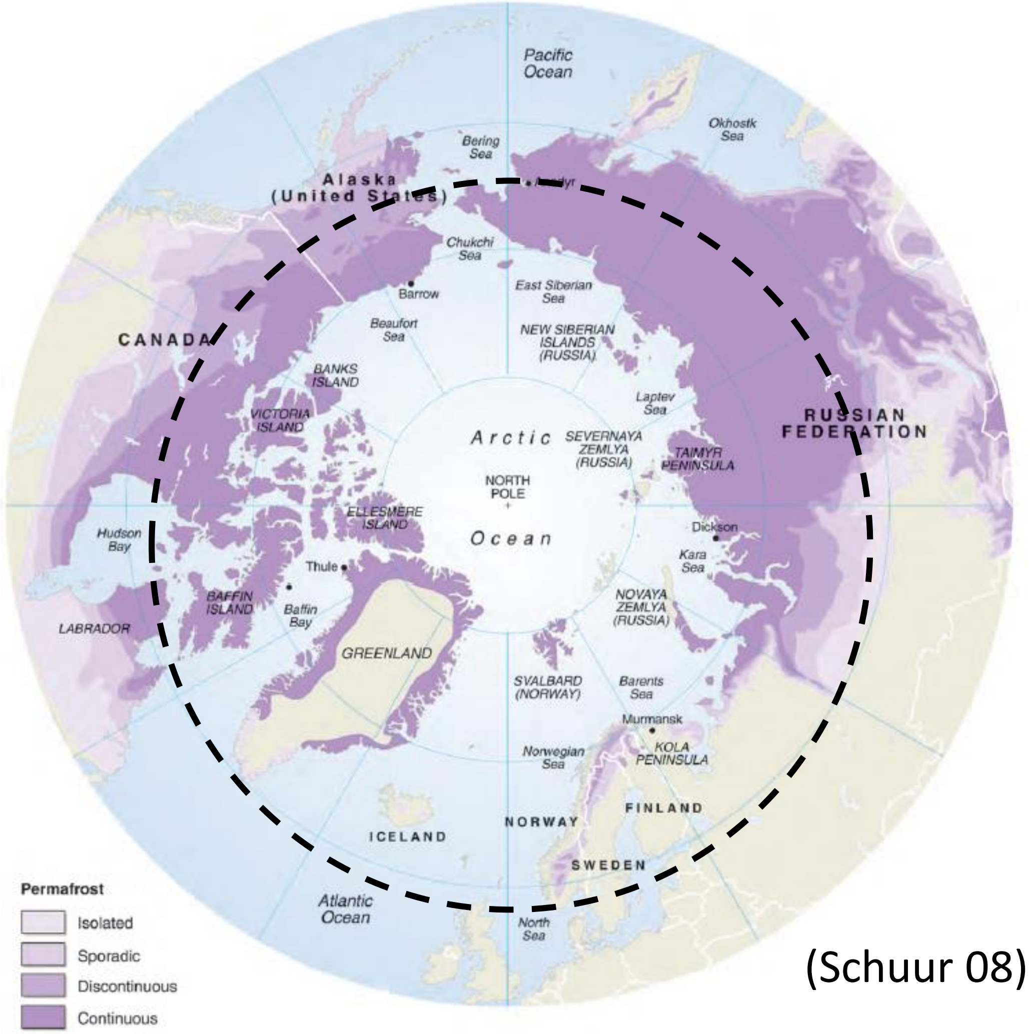
This is approximately what we would expect from a linear estimation of the permafrost line

Without permafrost amplification

Using a linear approximation to the permafrost line, Zebrowski and Nguyen estimated the permafrost line and increases in greenhouse gases due to permafrost.



(Zebrowski and Nguyen, *in prep*)



(Schuur 08)

With permafrost amplification